

ESTIMATED SHRINKAGE OF ONE-STAGE OF COBB-DOUGLAS PRODUCTION FUNCTION

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Abstract: This study aims at calculating the amount of bias for the estimated shrinkage ($\tilde{\beta}$) and the estimated shrinkage ($\tilde{\gamma}$), calculating the Mean of Squares Error (MSE) for estimator shrinkage ($\tilde{\beta}, \tilde{\gamma}$), and calculating the Relative Efficiency (RE) for the estimated ($\tilde{\beta}$) and ($\tilde{\gamma}$) with respect to the two estimated $\tilde{\beta}$ and the $\tilde{\gamma}$ and finally finding K value that makes $MSE(\tilde{\beta})$ and $MSE(\tilde{\gamma})$ minimization, the study find that the relative efficiency for the estimated ($\tilde{\beta}, \tilde{\gamma}$) have a high relative efficiency with respect to the mixed estimation method, and it suggests a way to produced the lowest Mean Squares Error, the study gives a the lowest MSE and the biggest relative compared with the mixed estimation method.

Key Words: Estimated Shrinkage ($\tilde{\beta}, \tilde{\gamma}$), Mean Square Error, Mixed Estimation, One-stage of Cobb-Douglas Production Function, Relative Efficiency.

INTRODUCTION

All research in the production functions based on the proposition that the production operations can be made better using the a homogenous linear function and by the elasticity of replacing some of the elements, this is the first trial in 1916 don by the economic (K.Wickesll)⁽⁶⁾ who use the mathematical function that represent the relationship between inputs and outputs. This function takes the name when the mathematical scientist (Coob 1928) and the economic scientist (Douglas 1928).

David Dumad (1937) precedes the application of the formula which was created by Zellner ,A.(1961, p.) and Wu, DE-MIN (1975) :

$$Q_t = \beta_0 L_t^{\beta_1} K_t^{\beta_2} e^{ut} \quad \dots (1)$$

Set:

Q_t : The amount or the value of the production in a specific period.

L_t : The labor measured by the average of the of workers in a specific period

K_t : The fixed capital measured by the total value of the fixed asset in a specific period

β_0 : Efficient Coefficient

β_1 : The production elasticity with proportion to the labor element

β_2 : The production elasticity with proportion to the capital element

u_t : The random error that distributed normally with a zero mean and σ_u^2 variance.

The estimated shrinkage with one stage calculated by the following formula^(4,5):

$$\tilde{\theta} = \psi(\hat{\theta})\hat{\theta} + [1 - \psi(\hat{\theta})]\theta_0 \quad \dots (2)$$

$\hat{\theta}$: An estimated calculated from a small sample by one of the classical ways

θ_0 : Represent prior information about θ parameter which must estimated as Initial Value

$\psi(\hat{\theta})$: A balanced shrinkage function can be constant or variable $0 \leq \psi(\hat{\theta}) \leq 1$

$\tilde{\theta}$: An estimated shrinkage has one stage and it is a linear formula contains the initial information θ_0 and the classical estimator $\hat{\theta}$.

Objective of the study: The study aims to calculating the following:

1. Calculating the amount of bias for the estimated shrinkage ($\tilde{\beta}$).
2. Calculating the amount of bias for the estimated shrinkage ($\tilde{\gamma}$).
3. Calculating the mean error squares for estimator shrinkage ($\tilde{\beta}$).
4. Calculating the mean error squares for estimator shrinkage ($\tilde{\gamma}$).
5. Calculating the relative efficient for the estimated ($\tilde{\beta}$) and ($\tilde{\gamma}$) with respect to the two estimated $\tilde{\beta}$ and the $\tilde{\gamma}$ which can be calculated by (RLS) method .
6. Finding K value that make $MSE(\tilde{\beta})$ and $MSE(\tilde{\gamma})$ at the least.

The mathematical explanation:

If the estimated shrinkage functions $\psi(\hat{\theta})$ fixed functions have amount K, which $0 \leq K \leq 1$ then:

$$\tilde{\theta} = K\hat{\theta} + (1 - K)\theta_0 \quad \dots (3)$$

Set:

K: Represents the balanced fixed shrinkage function, it's amount of confidence from the classical estimated $\hat{\theta}$, (K-1) represent the amount of confidence from the prior information θ_0 for the parameter θ which mean^(4,5) :

$$1 - \psi(\hat{\theta}) = 1 - K \quad , \quad \psi(\hat{\theta}) = K$$

For estimating the function parameter (β_1 and β_2) it must convert the (1) function to the leaner form by finding (Ln) for both function sides:

$$\ln Q_t = \ln(\beta_0) + \beta_1 \ln L_t + \beta_2 \ln K_t + u_t \quad \dots (4)$$

Propose:

$$\ln \beta_0 = \ln \alpha \quad , \quad \beta_1 = \beta \quad , \quad \beta_2 = \gamma$$

The function (4) can be written as the following:

$$\ln Q_t = \ln \alpha + \beta \ln L_t + \gamma \ln K_t + u_t \quad \dots (5)$$

The amount of shrinkage with one stage for β parameter as the following:

$$\tilde{\beta} = K\hat{\beta} + (1 - K)\beta_0 \quad \dots (6)$$

$$\tilde{\beta} = K(\hat{\beta} - \beta_0) + \beta_0 \quad \dots (7)$$

The solution steps as the following:

1. Estimating regression model parameter β and γ using the ordinary least squares (OLS) and adopting it as an initial values (γ_0, β_0) .
2. Estimating regression model parameter β and γ using the mixed estimation method (MEM) and adopting it as initial values (γ_0, β_0) .

Mixed Estimation Metho (MEM) :

The Mixed Estimation Method is one of the restricted regression models which will be explained to employee the initial information is an Inequality Restriction. This way include the combination between the initial information or that devised from out the sample and it provided from the secondary data or from the economic theories. The most important thing in the (MEM)^(9,10) use is that it is primary or the spatial information gives a specific range that specified by the biggest probability that include the real value of the requested parameter, taking that this range is compatible with the economic theory kind for the studied phenomenon ⁽⁸⁾ :

The estimation wave (b_{mem}) can be calculated as the following ^(1, 2, 3):

$$b_{mem} = [\sigma^{-2}X^T X + R^T G^{-1} R]^{-1} [\sigma^{-2}X^T Y + R^T G^{-1} r] \quad \dots (8)$$

Set:

r : a known vector has the $(j \times 1)$ rank.

R : constrains matrix has the $(j \times (k + 1))$ rank.

β : The parameter vector that has $[(k + 1) \times 1]$ rank.

V : The random error vector has $(j \times 1)$ rank.

And:

$$E(v) = 0 \quad , \quad E(vv^T) = G$$

Set:

G : The variance and the combined variance matrix for the previous estimators.

To explains that we suppose the previous information include tow unbiased estimators (b_1, b_2) for (β_1, β_2) respectively, if the variance of this estimators S_{11}, S_{22} and the combined variance S_{12} , it will be as the following^(1,2,37,11) :

$$r = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} , R = \begin{bmatrix} 1 & 0 & 0 & . & . & 0 \\ 0 & 1 & 0 & . & . & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix}$$

1. Calculating one stage shrinkage estimator using a fixed shrinkage function as the following:

$$\tilde{\beta} = K(\hat{\beta} - \beta_0) + \beta_0 \quad \dots (9)$$

$$\tilde{\gamma} = K(\hat{\gamma} - \gamma_0) + \gamma_0 \quad \dots (10)$$

Set:

$$\hat{\beta} \sim N[\beta, \text{Var}(\hat{\beta})]$$

$$\hat{\gamma} \sim N[\gamma, \text{Var}(\hat{\gamma})]$$

For calculating the bias of estimated shrinkage $\tilde{\beta}$:

$$\text{Bias}(\tilde{\beta}) = E(\tilde{\beta} - \beta)$$

$$\text{Bias}(\tilde{\beta}) = E[K(\hat{\beta} - \beta_0) + \beta_0 - \beta]$$

$$\text{Bias}(\tilde{\beta}) = K E(\hat{\beta} - \beta_0) - E(\beta - \beta_0)$$

$$= K \int_{-\infty}^{\infty} (\hat{\beta} - \beta_0) f(\hat{\beta}) d\hat{\beta} - \int_{-\infty}^{\infty} (\beta - \beta_0) f(\hat{\beta}) d\hat{\beta} \quad \dots (11)$$

$$= K \int_{-\beta}^{\infty} (\hat{\beta} - \beta_0) \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} \\ - \int_{-\infty}^{\infty} (\beta - \beta_0) \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} \quad \dots (12)$$

$$= K \int_{-\infty}^{\infty} (\hat{\beta} - \beta + \beta - \beta_0) \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} \\ - \int_{-\infty}^{\infty} (\beta - \beta_0) \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} \quad \dots (13)$$

$$= K \int_{-\infty}^{\infty} (\hat{\beta} - \beta) \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} + K \int_{-\infty}^{\infty} (\beta - \beta_0) \\ \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} - \int_{-\infty}^{\infty} (\beta - \beta_0) \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \\ \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} \quad \dots (14)$$

$$= K \int_{-\infty}^{\infty} (\hat{\beta} - \beta) \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} + (K - 1) \int_{-\infty}^{\infty} (\beta - \beta_0) \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \\ \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} \quad \dots (15)$$

Suppose:

$$Z = \frac{\hat{\beta} - \beta}{\sqrt{\text{Var}(\hat{\beta})}}, Z \sim N(0,1)$$

Then:

$$Z \sqrt{\text{Var}(\hat{\beta})} + \beta = \hat{\beta}$$

$$\sqrt{\text{Var}(\hat{\beta})} dZ = d\hat{\beta}$$

Suppose:

$$\lambda = \frac{\beta - \beta_0}{\sqrt{\text{Var}(\hat{\beta})}}$$

Then:

$$\text{Bias}(\tilde{\beta}) = K \sqrt{\text{Var}(\hat{\beta})} \int_{-\infty}^{\infty} \frac{(\hat{\beta} - \beta)}{\sqrt{\text{Var}(\hat{\beta})}} \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} +$$

$$(K - 1) \sqrt{\text{Var}(\hat{\beta})} \int_{-\infty}^{\infty} \frac{(\beta - \beta_0)}{\sqrt{\text{Var}(\hat{\beta})}} \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta} \quad \dots(16)$$

$$\text{Bias}(\tilde{\beta}) = K \sqrt{\text{Var}(\hat{\beta})} \int_{-\infty}^{\infty} Z \frac{1}{\sqrt{2\pi} \sqrt{\text{Var}(\hat{\beta})}} \text{Exp} \left[-Z^{2/2} \right] \sqrt{\text{Var}(\hat{\beta})} dZ +$$

$$(K - 1) \sqrt{\text{Var}(\hat{\beta})} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \sqrt{\text{Var}(\hat{\beta})}} \text{Exp} \left[-Z^{2/2} \right] \sqrt{\text{Var}(\hat{\beta})} dZ \quad \dots(17)$$

$$\text{Bias}(\tilde{\beta}) = K \sqrt{\text{Var}(\hat{\beta})} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} Z e^{-Z^{2/2}} dZ + (K - 1) \lambda \sqrt{\text{Var}(\hat{\beta})} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-Z^{2/2}} dZ \quad \dots(18)$$

$$\text{Bias}(\tilde{\beta}) = K \sqrt{\text{Var}(\hat{\beta})} E(Z) + (K - 1) \lambda \sqrt{\text{Var}(\hat{\beta})}$$

$$\text{Bias}(\tilde{\beta}) = (K - 1) \lambda \sqrt{\text{Var}(\hat{\beta})} \quad \dots (19)$$

By the same way the bias for estimator $\tilde{\gamma}$:

$$\text{Bias}(\tilde{\gamma}) = (K - 1) \lambda \sqrt{\text{Var}(\hat{\gamma})} \quad \dots (20)$$

- To calculate the mean error squares for estimator shrinkage ($\tilde{\beta}$)

$$\text{MSE}(\tilde{\beta}) = E(\tilde{\beta} - \beta)^2$$

$$\text{MSE}(\tilde{\beta}) = E \left[K(\hat{\beta} - \beta_0) + \beta_0 - \beta \right]^2$$

$$\text{MSE}(\tilde{\beta}) = E \left[K(\hat{\beta} - \beta_0) - (\beta - \beta_0) \right]^2$$

$$\text{MSE}(\tilde{\beta}) = K^2 E(\hat{\beta} - \beta_0)^2 - 2KE(\hat{\beta} - \beta_0)(\beta - \beta_0) + E(\beta - \beta_0)^2 \quad \dots (21)$$

$$MSE(\tilde{\beta}) = K^2 E(\hat{\beta} - \beta + \beta - \beta_0)^2 - 2K(\beta - \beta_0)E(\hat{\beta} - \beta + \beta - \beta_0) + E(\beta - \beta_0)^2$$

$$MSE(\tilde{\beta}) = K E(\hat{\beta} - \beta)^2 + K^2 E(\beta - \beta_0)^2 + 2K^2 E(\hat{\beta} - \beta)(\beta - \beta_0)$$

$$- 2K(\beta - \beta_0)E(\hat{\beta} - \beta) - 2KE(\beta - \beta_0)^2 + E(\beta - \beta_0)^2$$

$$MSE(\tilde{\beta}) = (K^2 - 2K + 1)E(\beta - \beta_0)^2 + (2K^2 - 2K)(\beta - \beta_0).E(\hat{\beta} - \beta) + K^2 E(\hat{\beta} - \beta)^2$$

$$MSE(\tilde{\beta}) = (K - 1)^2 E(\beta - \beta_0)^2 + 2K(K - 1)(\beta - \beta_0)E(\hat{\beta} - \beta) + K^2 E(\hat{\beta} - \beta)^2$$

$$E(\beta - \beta_0)^2 = \int_{-\infty}^{\infty} (\beta - \beta_0)^2 f(\hat{\beta}) d\hat{\beta} \quad \dots (22)$$

$$E(\beta - \beta_0)^2 = \int_{-\infty}^{\infty} (\beta - \beta_0)^2 \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp}\left[\frac{-(\hat{\beta} - \beta)^2}{-2\text{Var}(\hat{\beta})}\right] d\hat{\beta} \quad \dots (23)$$

$$E(\beta - \beta_0)^2 = \text{Var}(\hat{\beta}) \int_{-\infty}^{\infty} \frac{(\beta - \beta_0)^2}{\text{Var}(\hat{\beta})} \cdot \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp}\left[\frac{-(\hat{\beta} - \beta)^2}{-2\text{Var}(\hat{\beta})}\right] d\hat{\beta}$$

$$E(\beta - \beta_0)^2 = \text{Var}(\hat{\beta}) \int_{-\infty}^{\infty} \lambda^2 \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\text{Var}(\hat{\beta})}} e^{-Z^{2/2}} \sqrt{\text{Var}(\hat{\beta})} dZ$$

$$E(\beta - \beta_0)^2 = \text{Var}(\hat{\beta}) \lambda^2 \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-Z^{2/2}} dZ$$

$$E(\beta - \beta_0)^2 = \lambda^2 \text{Var}(\hat{\beta}) \quad \dots (24)$$

$$E(\hat{\beta} - \beta)(\beta - \beta_0) = \int_{-\infty}^{\infty} (\hat{\beta} - \beta)(\beta - \beta_0) f(\hat{\beta}) d\hat{\beta} \quad \dots (25)$$

$$E(\hat{\beta} - \beta)(\beta - \beta_0) = \int_{-\infty}^{\infty} (\hat{\beta} - \beta)(\beta - \beta_0) \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp}\left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})}\right] d\hat{\beta}$$

$$E(\hat{\beta} - \beta)(\beta - \beta_0) = \text{Var}(\hat{\beta}) \int_{-\infty}^{\infty} \frac{(\hat{\beta} - \beta)}{\sqrt{\text{Var}(\hat{\beta})}} \frac{(\beta - \beta_0)}{\sqrt{\text{Var}(\hat{\beta})}} \frac{1}{\sqrt{2\pi} \sqrt{\text{Var}(\hat{\beta})}} \text{Exp}\left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})}\right] d\hat{\beta}$$

$$E(\hat{\beta} - \beta)(\beta - \beta_0) = \text{Var}(\hat{\beta}) \int_{-\infty}^{\infty} Z\lambda \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\text{Var}(\hat{\beta})}} e^{-Z^{2/2}} \sqrt{\text{Var}(\hat{\beta})} dZ$$

$$E(\hat{\beta} - \beta)(\beta - \beta_0) = \lambda \text{Var}(\hat{\beta}) \int_{-\infty}^{\infty} Z \frac{1}{\sqrt{2\pi}} e^{-Z^{2/2}} dZ$$

$$E(\hat{\beta} - \beta)(\beta - \beta_0) = \lambda \text{Var}(\hat{\beta}) E(Z) = \text{Zero} \quad \dots (26)$$

$$E(\hat{\beta} - \beta)^2 = \int_{-\infty}^{\infty} (\hat{\beta} - \beta)^2 f(\hat{\beta}) d\hat{\beta}$$

$$E(\hat{\beta} - \beta)^2 = \int_{-\infty}^{\infty} (\hat{\beta} - \beta)^2 \frac{1}{\sqrt{2\pi \text{Var}(\hat{\beta})}} \text{Exp}\left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})}\right] d\hat{\beta} \quad \dots (27)$$

$$E(\hat{\beta} - \beta)^2 = \text{Var}(\hat{\beta}) \int_{-\infty}^{\infty} \frac{(\hat{\beta} - \beta)^2}{\text{Var}(\hat{\beta})} \frac{1}{\sqrt{2\pi} \sqrt{\text{Var}(\hat{\beta})}} \text{Exp} \left[\frac{-(\hat{\beta} - \beta)^2}{2\text{Var}(\hat{\beta})} \right] d\hat{\beta}$$

$$E(\hat{\beta} - \beta)^2 = \text{Var}(\hat{\beta}) \int_{-\infty}^{\infty} Z^2 \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\text{Var}(\hat{\beta})}} e^{-Z^2/2} \sqrt{\text{Var}(\hat{\beta})} dZ$$

$$E(\hat{\beta} - \beta)^2 = \text{Var}(\hat{\beta}) \int_{-\infty}^{\infty} Z^2 \frac{1}{\sqrt{2\pi}} e^{-Z^2/2} dZ$$

$$E(\hat{\beta} - \beta)^2 = \text{Var}(\hat{\beta}) E(Z^2) = \text{Var}(\hat{\beta}) \dots (28)$$

$$MSE(\tilde{\beta}) = (K - 1)^2 \lambda^2 \text{Var}(\hat{\beta}) + 2K(K - 1)(\text{Zero}) + K^2 \text{Var}(\hat{\beta})$$

$$MSE(\tilde{\beta}) = (K - 1)^2 \lambda^2 \text{Var}(\hat{\beta}) + K^2 \text{Var}(\hat{\beta})$$

$$MSE(\tilde{\beta}) = \text{Var}(\hat{\beta}) \left[(K - 1)^2 \lambda^2 + K^2 \right] \dots (29)$$

By the same way the mean error squares for estimator shrinkage ($\tilde{\gamma}$) can be yielded by the following function:

$$MSE(\tilde{\gamma}) = \text{Var}(\tilde{\gamma}) \left[(K - 1)^2 \lambda^2 + K^2 \right] \dots (30)$$

The relative efficient for the estimated $\tilde{\beta}$ and $\tilde{\gamma}$ for the estimators $\hat{\beta}$ and $\hat{\gamma}$ which can be calculated by

Restricted Least Square method (RLS):

$$R.E(\tilde{\beta}) = \frac{MSE(\hat{\beta})}{MSE(\tilde{\beta})}$$

$$R.E(\tilde{\beta}) = \frac{\text{Var}(\hat{\beta})}{\text{Var}(\hat{\beta}) \left[(K - 1)^2 \lambda^2 + K^2 \right]}$$

$$R.E(\tilde{\beta}) = \left[(K - 1)^2 \lambda^2 + K^2 \right]^{-1} \dots (31)$$

$$R.E(\tilde{\gamma}) = \frac{MSE(\hat{\gamma})}{MSE(\tilde{\gamma})}$$

$$R.E(\tilde{\gamma}) = \frac{\text{Var}(\hat{\gamma})}{\text{Var}(\tilde{\gamma}) \left[(K - 1)^2 \lambda^2 + K^2 \right]}$$

$$R.E(\tilde{\gamma}) = \left[(K - 1)^2 \lambda^2 + K^2 \right]^{-1} \dots (32)$$

To calculate K value that make $MSE(\tilde{\beta})$ and $MSE(\tilde{\gamma})$ the least, the first derivative for the mean error squares calculated for K and equaling it to 0:

$$\frac{\partial MSE(\tilde{\beta})}{\partial K} = 2(K - 1)\lambda^2 \text{Var}(\hat{\beta}) + 2K \text{Var}(\hat{\beta})$$

$$\frac{\partial MSE(\tilde{\beta})}{\partial K} = 0$$

$$\Rightarrow 2(K-1)\lambda^2 \text{Var}(\hat{\beta}) + 2K \text{Var}(\hat{\beta}) = 0$$

$$K\lambda^2 \text{Var}(\hat{\beta}) + K \text{Var}(\hat{\beta}) = \lambda^2 \text{Var}(\hat{\beta})$$

$$[K\lambda^2 + K] \text{Var}(\hat{\beta}) = \lambda^2 \text{Var}(\hat{\beta})$$

$$K(\lambda^2 + 1) = \lambda^2$$

$$K = \frac{\lambda^2}{\lambda^2 + 1} \quad \dots (33)$$

$$\frac{\partial^2 \text{MSE}(\tilde{\beta})}{\partial K^2} = 2\lambda^2 \text{Var}(\hat{\beta}) + 2 \text{Var}(\hat{\beta}) > 0$$

Is minimization

RESULTS AND RECOMMENDATION

Table (1) and (2) show the following:

1. The unbiased values for the one stage shrinkage estimator $(\tilde{\beta}, \tilde{\gamma})$ increase and then start decreasing until the real value of the parameter equals or closed to equal the initial value.
2. The mean square error (MSE) for the one stage shrinkage estimator $(\tilde{\beta}, \tilde{\gamma})$ decrease until the real value of parameter equals or closed to initial value and the mean standard error increase as a result the parameter real value shifting away from the initial value.
3. The proposed real efficient (R.E) for the one stage shrinkage estimator $(\tilde{\beta}, \tilde{\gamma})$ have a high proportion efficiency for the mixed estimation method, and the proposed method results the lowest mean square error and the highest relative efficiency compared with the mixed estimation method.
4. The study recommends the use of one stage shrinkage estimator in the Coob-Douglas functions for calculating the utilization proportion of labor and the fixed capital percent in the production functions analysis generally.

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APPENDIX

Table 1: The estimator ($\tilde{\beta}$), the BIAS ($\tilde{\beta}$), the M.S.E ($\tilde{\beta}$), R.E ($\tilde{\beta}$)

β	B_0	$\hat{\beta}$	S.E($\hat{\beta}$)	λ	K	$\tilde{\beta}$	BIAS($\tilde{\beta}$)	M.S.E($\tilde{\beta}$)	R.E($\tilde{\beta}$)
0.01	0.677	0.523	0.2317	-2.87872	0.89232	0.53958	0.07182	0.047904	1.12067
0.02	0.677	0.523	0.2317	-2.83556	0.88939	0.54003	0.072674	0.047747	1.124372
0.03	0.677	0.523	0.2317	-2.7924	0.88633	0.5405	0.073543	0.047583	1.128246
0.04	0.677	0.523	0.2317	-2.74924	0.88315	0.54099	0.07443	0.047412	1.132304
0.05	0.677	0.523	0.2317	-2.70609	0.87985	0.5415	0.075334	0.047235	1.136558
0.06	0.677	0.523	0.2317	-2.66293	0.87641	0.54203	0.076256	0.04705	1.14102
0.07	0.677	0.523	0.2317	-2.61977	0.87282	0.54258	0.077195	0.046858	1.145705
0.08	0.677	0.523	0.2317	-2.57661	0.86909	0.54316	0.078153	0.046657	1.150627
0.09	0.677	0.523	0.2317	-2.53345	0.8652	0.54376	0.079128	0.046448	1.155803
0.1	0.677	0.523	0.2317	-2.49029	0.86114	0.54438	0.080122	0.04623	1.16125
0.11	0.677	0.523	0.2317	-2.44713	0.85691	0.54504	0.081134	0.046003	1.166988
0.12	0.677	0.523	0.2317	-2.40397	0.85249	0.54572	0.082165	0.045766	1.173038
0.13	0.677	0.523	0.2317	-2.36081	0.84787	0.54643	0.083214	0.045518	1.179423
0.14	0.677	0.523	0.2317	-2.31765	0.84305	0.54717	0.084281	0.045259	1.186167
0.15	0.677	0.523	0.2317	-2.27449	0.83801	0.54795	0.085367	0.044989	1.1933
0.16	0.677	0.523	0.2317	-2.23133	0.83274	0.54876	0.086471	0.044706	1.20085
0.17	0.677	0.523	0.2317	-2.18817	0.82723	0.54961	0.087593	0.04441	1.208851
0.18	0.677	0.523	0.2317	-2.14502	0.82146	0.55049	0.088733	0.0441	1.21734
0.19	0.677	0.523	0.2317	-2.10186	0.81542	0.55142	0.089889	0.043776	1.226357
0.2	0.677	0.523	0.2317	-2.0587	0.8091	0.5524	0.091061	0.043436	1.235947
0.21	0.677	0.523	0.2317	-2.01554	0.80246	0.55342	0.092249	0.04308	1.24616
0.22	0.677	0.523	0.2317	-1.97238	0.79551	0.55449	0.093451	0.042707	1.257051
0.23	0.677	0.523	0.2317	-1.92922	0.78822	0.55561	0.094666	0.042316	1.268681
0.24	0.677	0.523	0.2317	-1.88606	0.78057	0.55679	0.095892	0.041905	1.281118

0.25	0.677	0.523	0.2317	-1.8429	0.77254	0.55803	0.097128	0.041473	1.29444
0.26	0.677	0.523	0.2317	-1.79974	0.7641	0.55933	0.098371	0.041021	1.308731
0.27	0.677	0.523	0.2317	-1.75658	0.75524	0.56069	0.099619	0.040545	1.324088
0.28	0.677	0.523	0.2317	-1.71342	0.74592	0.56213	0.100869	0.040045	1.340621
0.29	0.677	0.523	0.2317	-1.67026	0.73613	0.56364	0.102117	0.039519	1.358451
0.3	0.677	0.523	0.2317	-1.6271	0.72584	0.56522	0.103359	0.038966	1.377719
0.31	0.677	0.523	0.2317	-1.58394	0.71501	0.56689	0.104592	0.038385	1.398584
0.32	0.677	0.523	0.2317	-1.54079	0.70362	0.56864	0.105809	0.037774	1.421226
0.33	0.677	0.523	0.2317	-1.49763	0.69163	0.57049	0.107004	0.03713	1.445854

β	B_0	$\hat{\beta}$	S.E($\hat{\beta}$)	λ	K	$\tilde{\beta}$	BIAS($\tilde{\beta}$)	M.S.E($\tilde{\beta}$)	R.E($\tilde{\beta}$)
0.34	0.677	0.523	0.2317	-1.45447	0.67902	0.57243	0.10817	0.036453	1.472707
0.35	0.677	0.523	0.2317	-1.41131	0.66575	0.57447	0.109299	0.035741	1.502061
0.36	0.677	0.523	0.2317	-1.36815	0.65179	0.57662	0.110383	0.034991	1.534236
0.37	0.677	0.523	0.2317	-1.32499	0.6371	0.57889	0.11141	0.034203	1.569607
0.38	0.677	0.523	0.2317	-1.28183	0.62165	0.58127	0.112369	0.033373	1.60861
0.39	0.677	0.523	0.2317	-1.23867	0.60541	0.58377	0.113246	0.032502	1.651761
0.4	0.677	0.523	0.2317	-1.19551	0.58835	0.58639	0.114027	0.031586	1.699669
0.41	0.677	0.523	0.2317	-1.15235	0.57043	0.58915	0.114695	0.030624	1.75306
0.42	0.677	0.523	0.2317	-1.10919	0.55163	0.59205	0.115231	0.029614	1.812804
0.43	0.677	0.523	0.2317	-1.06603	0.53193	0.59508	0.115614	0.028557	1.87995
0.44	0.677	0.523	0.2317	-1.02287	0.51131	0.59826	0.11582	0.027449	1.955774
0.45	0.677	0.523	0.2317	-0.97972	0.48975	0.60158	0.115826	0.026292	2.041838
0.46	0.677	0.523	0.2317	-0.93656	0.46727	0.60504	0.115602	0.025086	2.140073
0.47	0.677	0.523	0.2317	-0.8934	0.44388	0.60864	0.115118	0.023829	2.252885
0.48	0.677	0.523	0.2317	-0.85024	0.41958	0.61238	0.114342	0.022525	2.38331
0.49	0.677	0.523	0.2317	-0.80708	0.39444	0.61626	0.113239	0.021176	2.535214
0.5	0.677	0.523	0.2317	-0.76392	0.36852	0.62025	0.111773	0.019784	2.713585
0.51	0.677	0.523	0.2317	-0.72076	0.34189	0.62435	0.109905	0.018354	2.924949
0.52	0.677	0.523	0.2317	-0.6776	0.31467	0.62854	0.107597	0.016893	3.177974
0.53	0.677	0.523	0.2317	-0.63444	0.287	0.6328	0.104812	0.015407	3.484376
0.54	0.677	0.523	0.2317	-0.59128	0.25905	0.63711	0.10151	0.013907	3.860296
0.55	0.677	0.523	0.2317	-0.54812	0.23103	0.64142	0.097659	0.012403	4.32847
0.56	0.677	0.523	0.2317	-0.50496	0.20318	0.64571	0.093228	0.010908	4.921754

0.57	0.677	0.523	0.2317	-0.4618	0.17578	0.64993	0.088192	0.009437	5.689046
0.58	0.677	0.523	0.2317	-0.41865	0.14913	0.65403	0.082535	0.008006	6.705696
0.59	0.677	0.523	0.2317	-0.37549	0.12357	0.65797	0.07625	0.006634	8.092732
0.6	0.677	0.523	0.2317	-0.33233	0.09946	0.66168	0.069342	0.005339	10.05463
0.61	0.677	0.523	0.2317	-0.28917	0.07717	0.66512	0.06183	0.004143	12.95921
0.62	0.677	0.523	0.2317	-0.24601	0.05707	0.66821	0.053747	0.003064	17.52351
0.63	0.677	0.523	0.2317	-0.20285	0.03952	0.67091	0.045143	0.002122	25.3028
0.64	0.677	0.523	0.2317	-0.15969	0.02487	0.67317	0.03608	0.001335	40.21467
0.65	0.677	0.523	0.2317	-0.11653	0.0134	0.67494	0.026638	0.000719	74.64182
0.66	0.677	0.523	0.2317	-0.07337	0.00535	0.67618	0.016909	0.000287	186.7609

B	B ₀	$\hat{\beta}$	S.E($\hat{\beta}$)	λ	K	$\tilde{\beta}$	BIAS($\tilde{\beta}$)	M.S.E($\tilde{\beta}$)	R.E($\tilde{\beta}$)
0.67	0.677	0.523	0.2317	-0.03021	0.00091	0.67686	0.006994	4.9E-05	1096.61
0.68	0.677	0.523	0.2317	0.012948	0.00017	0.67697	-0.003	0.000009	5965.988
0.69	0.677	0.523	0.2317	0.056107	0.00314	0.67652	-0.01296	0.000168	318.6621
0.7	0.677	0.523	0.2317	0.099266	0.00976	0.6755	-0.02278	0.000524	102.4837
0.71	0.677	0.523	0.2317	0.142426	0.01988	0.67394	-0.03234	0.001067	50.29742
0.72	0.677	0.523	0.2317	0.185585	0.03329	0.67187	-0.04157	0.001787	30.03455
0.73	0.677	0.523	0.2317	0.228744	0.04972	0.66934	-0.05036	0.002669	20.11174
0.74	0.677	0.523	0.2317	0.271903	0.06884	0.6664	-0.05866	0.003696	14.52605
0.75	0.677	0.523	0.2317	0.315063	0.0903	0.66309	-0.06641	0.004848	11.0741
0.76	0.677	0.523	0.2317	0.358222	0.11373	0.65949	-0.07356	0.006106	8.792842
0.77	0.677	0.523	0.2317	0.401381	0.13875	0.65563	-0.0801	0.007449	7.207063
0.78	0.677	0.523	0.2317	0.44454	0.16501	0.65159	-0.086	0.008858	6.060316
0.79	0.677	0.523	0.2317	0.4877	0.19215	0.64741	-0.09129	0.010315	5.204314
0.8	0.677	0.523	0.2317	0.530859	0.21985	0.64314	-0.09596	0.011803	4.548476
0.81	0.677	0.523	0.2317	0.574018	0.24784	0.63883	-0.10004	0.013305	4.034931
0.82	0.677	0.523	0.2317	0.617177	0.27584	0.63452	-0.10356	0.014808	3.625306
0.83	0.677	0.523	0.2317	0.660337	0.30364	0.63024	-0.10654	0.016301	3.293344
0.84	0.677	0.523	0.2317	0.703496	0.33106	0.62602	-0.10904	0.017773	3.020584
0.85	0.677	0.523	0.2317	0.746655	0.35794	0.62188	-0.11108	0.019216	2.793742
0.86	0.677	0.523	0.2317	0.789814	0.38416	0.61784	-0.1127	0.020624	2.60306
0.87	0.677	0.523	0.2317	0.832974	0.40963	0.61392	-0.11394	0.021991	2.441244
0.88	0.677	0.523	0.2317	0.876133	0.43426	0.61012	-0.11484	0.023313	2.302747
0.89	0.677	0.523	0.2317	0.919292	0.45802	0.60646	-0.11544	0.024589	2.183295
0.9	0.677	0.523	0.2317	0.962451	0.48087	0.60295	-0.11577	0.025816	2.079549

0.91	0.677	0.523	0.2317	1.005611	0.5028	0.59957	-0.11585	0.026993	1.988872
0.92	0.677	0.523	0.2317	1.04877	0.52379	0.59634	-0.11572	0.02812	1.909158
0.93	0.677	0.523	0.2317	1.091929	0.54386	0.59325	-0.1154	0.029197	1.838708
0.94	0.677	0.523	0.2317	1.135088	0.56302	0.5903	-0.11493	0.030226	1.776141
0.95	0.677	0.523	0.2317	1.178248	0.58129	0.58748	-0.11431	0.031206	1.720322
0.96	0.677	0.523	0.2317	1.221407	0.59869	0.5848	-0.11357	0.032141	1.670315
0.97	0.677	0.523	0.2317	1.264566	0.61526	0.58225	-0.11273	0.03303	1.625341
0.98	0.677	0.523	0.2317	1.307726	0.63102	0.57982	-0.1118	0.033876	1.584745
0.99	0.677	0.523	0.2317	1.350885	0.646	0.57752	-0.1108	0.034681	1.547978

Table 2: The estimator ($\tilde{\gamma}$), the BIAS ($\tilde{\gamma}$), the MSE ($\tilde{\gamma}$) and R.E ($\tilde{\gamma}$)

γ	γ_0	$\hat{\gamma}$	S.E($\hat{\gamma}$)	λ	K	$\tilde{\gamma}$	BIAS($\tilde{\gamma}$)	MSE($\tilde{\gamma}$)	R.E($\tilde{\gamma}$)
0.01	0.48	0.507	0.0741828	-6.335700459	0.97569	0.506343722	0.0114241	0.0053693	1.02491
0.02	0.48	0.507	0.0741828	-6.200898321	0.97465	0.506315609	0.0116600	0.0053636	1.02601
0.03	0.48	0.507	0.0741828	-6.066096184	0.97354	0.506285667	0.0119055	0.0053575	1.02718
0.04	0.48	0.507	0.0741828	-5.931294047	0.97236	0.506253736	0.0121613	0.0053510	1.02843
0.05	0.48	0.507	0.0741828	-5.796491909	0.97110	0.506219638	0.0124280	0.0053440	1.02976
0.06	0.48	0.507	0.0741828	-5.661689772	0.96975	0.506183173	0.0127062	0.0053366	1.03120
0.07	0.48	0.507	0.0741828	-5.526887634	0.96830	0.506144120	0.0129967	0.0053286	1.03274
0.08	0.48	0.507	0.0741828	-5.392085497	0.96675	0.506102232	0.0133003	0.0053201	1.03439
0.09	0.48	0.507	0.0741828	-5.257283359	0.96508	0.506057231	0.0136178	0.0053109	1.03618
0.10	0.48	0.507	0.0741828	-5.122481222	0.96329	0.506008804	0.0139502	0.0053011	1.03811
0.11	0.48	0.507	0.0741828	-4.987679085	0.96136	0.505956600	0.0142984	0.0052904	1.04020
0.12	0.48	0.507	0.0741828	-4.852876947	0.95927	0.505900222	0.0146637	0.0052789	1.04246
0.13	0.48	0.507	0.0741828	-4.718074810	0.95701	0.505839220	0.0150471	0.0052665	1.04492
0.14	0.48	0.507	0.0741828	-4.583272672	0.95456	0.505773084	0.0154501	0.0052530	1.04760
0.15	0.48	0.507	0.0741828	-4.448470535	0.95190	0.505701229	0.0158739	0.0052384	1.05053
0.16	0.48	0.507	0.0741828	-4.313668398	0.94900	0.505622992	0.0163201	0.0052224	1.05374
0.17	0.48	0.507	0.0741828	-4.178866260	0.94584	0.505537610	0.0167904	0.0052050	1.05726
0.18	0.48	0.507	0.0741828	-4.044064123	0.94238	0.505444203	0.0172866	0.0051860	1.06115
0.19	0.48	0.507	0.0741828	-3.909261985	0.93858	0.505341761	0.0178107	0.0051651	1.06544
0.20	0.48	0.507	0.0741828	-3.774459848	0.93441	0.505229107	0.0183648	0.0051421	1.07019
0.21	0.48	0.507	0.0741828	-3.639657710	0.92981	0.505104879	0.0189512	0.0051168	1.07549
0.22	0.48	0.507	0.0741828	-3.504855573	0.92472	0.504967482	0.0195724	0.0050888	1.08141
0.23	0.48	0.507	0.0741828	-3.370053436	0.91908	0.504815050	0.0202310	0.0050578	1.08805
0.24	0.48	0.507	0.0741828	-3.235251298	0.91279	0.504645387	0.0209299	0.0050232	1.09554

0.25	0.48	0.507	0.0741828	-3.100449161	0.90577	0.504455899	0.0216720	0.0049846	1.10403
0.26	0.48	0.507	0.0741828	-2.965647023	0.89791	0.504243509	0.0224603	0.0049413	1.11370
0.27	0.48	0.507	0.0741828	-2.830844886	0.88906	0.504004554	0.0232979	0.0048926	1.12479
0.28	0.48	0.507	0.0741828	-2.696042748	0.87906	0.503734653	0.0241878	0.0048376	1.13758
0.29	0.48	0.507	0.0741828	-2.561240611	0.86772	0.503428549	0.0251324	0.0047752	1.15244
0.30	0.48	0.507	0.0741828	-2.426438474	0.85481	0.503079914	0.0261339	0.0047041	1.16985
0.31	0.48	0.507	0.0741828	-2.291636336	0.84004	0.502681104	0.0271931	0.0046228	1.19042
0.32	0.48	0.507	0.0741828	-2.156834199	0.82307	0.502222874	0.0283089	0.0045294	1.21496
0.33	0.48	0.507	0.0741828	-2.022032061	0.80348	0.501694036	0.0294776	0.0044216	1.24458

γ	γ_0	$\hat{\gamma}$	S.E($\hat{\gamma}$)	λ	K	$\tilde{\gamma}$	BIAS($\tilde{\gamma}$)	MSE($\tilde{\gamma}$)	R.E($\tilde{\gamma}$)
0.34	0.48	0.507	0.0741828	-1.887229924	0.78078	0.501081072	0.0306907	0.0042967	1.28077
0.35	0.48	0.507	0.0741828	-1.752427786	0.75436	0.500367728	0.0319332	0.0041513	1.32563
0.36	0.48	0.507	0.0741828	-1.617625649	0.72351	0.499534657	0.0331793	0.0039815	1.38216
0.37	0.48	0.507	0.0741828	-1.482823512	0.68738	0.498559244	0.0343883	0.0037827	1.45480
0.38	0.48	0.507	0.0741828	-1.348021374	0.64503	0.497415885	0.0354967	0.0035497	1.55031
0.39	0.48	0.507	0.0741828	-1.213219237	0.59545	0.496077232	0.0364092	0.0032768	1.67939
0.40	0.48	0.507	0.0741828	-1.078417099	0.53768	0.494517241	0.0369860	0.0029589	1.85986
0.41	0.48	0.507	0.0741828	-0.943614962	0.47101	0.492717378	0.0370290	0.0025920	2.12308
0.42	0.48	0.507	0.0741828	-0.808812825	0.39547	0.490677696	0.0362718	0.0021763	2.52864
0.43	0.48	0.507	0.0741828	-0.674010687	0.31238	0.488434245	0.0343810	0.0017191	3.20124
0.44	0.48	0.507	0.0741828	-0.539208550	0.22525	0.486081862	0.0309898	0.0012396	4.43943
0.45	0.48	0.507	0.0741828	-0.404406412	0.14056	0.483795044	0.0257833	0.0007735	7.11454
0.46	0.48	0.507	0.0741828	-0.269604275	0.06776	0.481829551	0.0186448	0.0003729	14.75772
0.47	0.48	0.507	0.0741828	-0.134802137	0.01785	0.480481877	0.0098215	0.0000982	56.03088
0.48	0.48	0.507	0.0741828	0.000000000	0.00000	0.480000000	0.0000000	0.0000000	#NULL!
0.49	0.48	0.507	0.0741828	0.134802137	0.01785	0.480481877	-0.0098215	0.0000982	56.03088
0.50	0.48	0.507	0.0741828	0.269604275	0.06776	0.481829551	-0.0186448	0.0003729	14.75772
0.51	0.48	0.507	0.0741828	0.404406412	0.14056	0.483795044	-0.0257833	0.0007735	7.11454
0.52	0.48	0.507	0.0741828	0.539208550	0.22525	0.486081862	-0.0309898	0.0012396	4.43943
0.53	0.48	0.507	0.0741828	0.674010687	0.31238	0.488434245	-0.0343810	0.0017191	3.20124
0.54	0.48	0.507	0.0741828	0.808812825	0.39547	0.490677696	-0.0362718	0.0021763	2.52864
0.55	0.48	0.507	0.0741828	0.943614962	0.47101	0.492717378	-0.0370290	0.0025920	2.12308
0.56	0.48	0.507	0.0741828	1.078417099	0.53768	0.494517241	-0.0369860	0.0029589	1.85986
0.57	0.48	0.507	0.0741828	1.213219237	0.59545	0.496077232	-0.0364092	0.0032768	1.67939
0.58	0.48	0.507	0.0741828	1.348021374	0.64503	0.497415885	-0.0354967	0.0035497	1.55031
0.59	0.48	0.507	0.0741828	1.482823512	0.68738	0.498559244	-0.0343883	0.0037827	1.45480
0.60	0.48	0.507	0.0741828	1.617625649	0.72351	0.499534657	-0.0331793	0.0039815	1.38216
0.61	0.48	0.507	0.0741828	1.752427786	0.75436	0.500367728	-0.0319332	0.0041513	1.32563
0.62	0.48	0.507	0.0741828	1.887229924	0.78078	0.501081072	-0.0306907	0.0042967	1.28077
0.63	0.48	0.507	0.0741828	2.022032061	0.80348	0.501694036	-0.0294776	0.0044216	1.24458
0.64	0.48	0.507	0.0741828	2.156834199	0.82307	0.502222874	-0.0283089	0.0045294	1.21496
0.65	0.48	0.507	0.0741828	2.291636336	0.84004	0.502681104	-0.0271931	0.0046228	1.19042
0.66	0.48	0.507	0.0741828	2.426438474	0.85481	0.503079914	-0.0261339	0.0047041	1.16985

γ	γ_0	$\hat{\gamma}$	S.E($\hat{\gamma}$)	λ	K	$\tilde{\gamma}$	BIAS($\tilde{\gamma}$)	MSE($\tilde{\gamma}$)	R.E($\tilde{\gamma}$)
0.67	0.48	0.507	0.0741828	2.561240611	0.86772	0.503428549	-0.0251324	0.0047752	1.15244
0.68	0.48	0.507	0.0741828	2.696042748	0.87906	0.503734653	-0.0241878	0.0048376	1.13758
0.69	0.48	0.507	0.0741828	2.830844886	0.88906	0.504004554	-0.0232979	0.0048926	1.12479
0.70	0.48	0.507	0.0741828	2.965647023	0.89791	0.504243509	-0.0224603	0.0049413	1.11370
0.71	0.48	0.507	0.0741828	3.100449161	0.90577	0.504455899	-0.0216720	0.0049846	1.10403
0.72	0.48	0.507	0.0741828	3.235251298	0.91279	0.504645387	-0.0209299	0.0050232	1.09554
0.73	0.48	0.507	0.0741828	3.370053436	0.91908	0.504815050	-0.0202310	0.0050578	1.08805
0.74	0.48	0.507	0.0741828	3.504855573	0.92472	0.504967482	-0.0195724	0.0050888	1.08141
0.75	0.48	0.507	0.0741828	3.639657710	0.92981	0.505104879	-0.0189512	0.0051168	1.07549
0.76	0.48	0.507	0.0741828	3.774459848	0.93441	0.505229107	-0.0183648	0.0051421	1.07019
0.77	0.48	0.507	0.0741828	3.909261985	0.93858	0.505341761	-0.0178107	0.0051651	1.06544
0.78	0.48	0.507	0.0741828	4.044064123	0.94238	0.505444203	-0.0172866	0.0051860	1.06115
0.79	0.48	0.507	0.0741828	4.178866260	0.94584	0.505537610	-0.0167904	0.0052050	1.05726
0.80	0.48	0.507	0.0741828	4.313668398	0.94900	0.505622992	-0.0163201	0.0052224	1.05374
0.81	0.48	0.507	0.0741828	4.448470535	0.95190	0.505701229	-0.0158739	0.0052384	1.05053
0.82	0.48	0.507	0.0741828	4.583272672	0.95456	0.505773084	-0.0154501	0.0052530	1.04760
0.83	0.48	0.507	0.0741828	4.718074810	0.95701	0.505839220	-0.0150471	0.0052665	1.04492
0.84	0.48	0.507	0.0741828	4.852876947	0.95927	0.505900222	-0.0146637	0.0052789	1.04246
0.85	0.48	0.507	0.0741828	4.987679085	0.96136	0.505956600	-0.0142984	0.0052904	1.04020
0.86	0.48	0.507	0.0741828	5.122481222	0.96329	0.506008804	-0.0139502	0.0053011	1.03811
0.87	0.48	0.507	0.0741828	5.257283359	0.96508	0.506057231	-0.0136178	0.0053109	1.03618
0.88	0.48	0.507	0.0741828	5.392085497	0.96675	0.506102232	-0.0133003	0.0053201	1.03439
0.89	0.48	0.507	0.0741828	5.526887634	0.96830	0.506144120	-0.0129967	0.0053286	1.03274
0.90	0.48	0.507	0.0741828	5.661689772	0.96975	0.506183173	-0.0127062	0.0053366	1.03120
0.91	0.48	0.507	0.0741828	5.796491909	0.97110	0.506219638	-0.0124280	0.0053440	1.02976
0.92	0.48	0.507	0.0741828	5.931294047	0.97236	0.506253736	-0.0121613	0.0053510	1.02843
0.93	0.48	0.507	0.0741828	6.066096184	0.97354	0.506285667	-0.0119055	0.0053575	1.02718
0.94	0.48	0.507	0.0741828	6.200898321	0.97465	0.506315609	-0.0116600	0.0053636	1.02601
0.95	0.48	0.507	0.0741828	6.335700459	0.97569	0.506343722	-0.0114241	0.0053693	1.02491
0.96	0.48	0.507	0.0741828	6.470502596	0.97667	0.506370151	-0.0111973	0.0053747	1.02388
0.97	0.48	0.507	0.0741828	6.605304734	0.97759	0.506395026	-0.0109791	0.0053798	1.02292
0.98	0.48	0.507	0.0741828	6.740106871	0.97846	0.506418467	-0.0107691	0.0053846	1.02201
0.99	0.48	0.507	0.0741828	6.874909009	0.97928	0.506440581	-0.0105668	0.0053891	1.02116